XS-Stabilizer Formalism

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joint work with Buerschaper, Van den Nest

QIP 15’

http://arxiv.org/abs/1404.5327

Outline

- Motivation
- Example: double semion model
- Summary of properties

Definition

- Pauli-S group: $\mathcal{P}_n^s = \langle \alpha, X, S \rangle^{\otimes n}$

$$\alpha = \sqrt{i} \quad S = \mathrm{diag}(1, i) \quad S^{-1} X S = -iXZ$$

$$X \otimes S \otimes Z$$

- Given $G = \langle g_1, \ldots, g_m \rangle \subset \mathcal{P}_n^s$

We call a state $|\psi\rangle$ XS-stabilizer state if (uniquely)

$$g_j |\psi\rangle = |\psi\rangle$$

When not unique, we call it XS-stabilizer code

Motivation

Pauli stabilizer formalism

- (Innocently looking) tensor product operators
- Most properties from commutation relation and linear algebra
- Numerous applications: Fault tolerance, measurement based computation, etc

XS stabilizer

- (Still innocently looking) tensor product operators
- Many properties from commutation relation and linear algebra
- Simple to learn

Toric (surface) code

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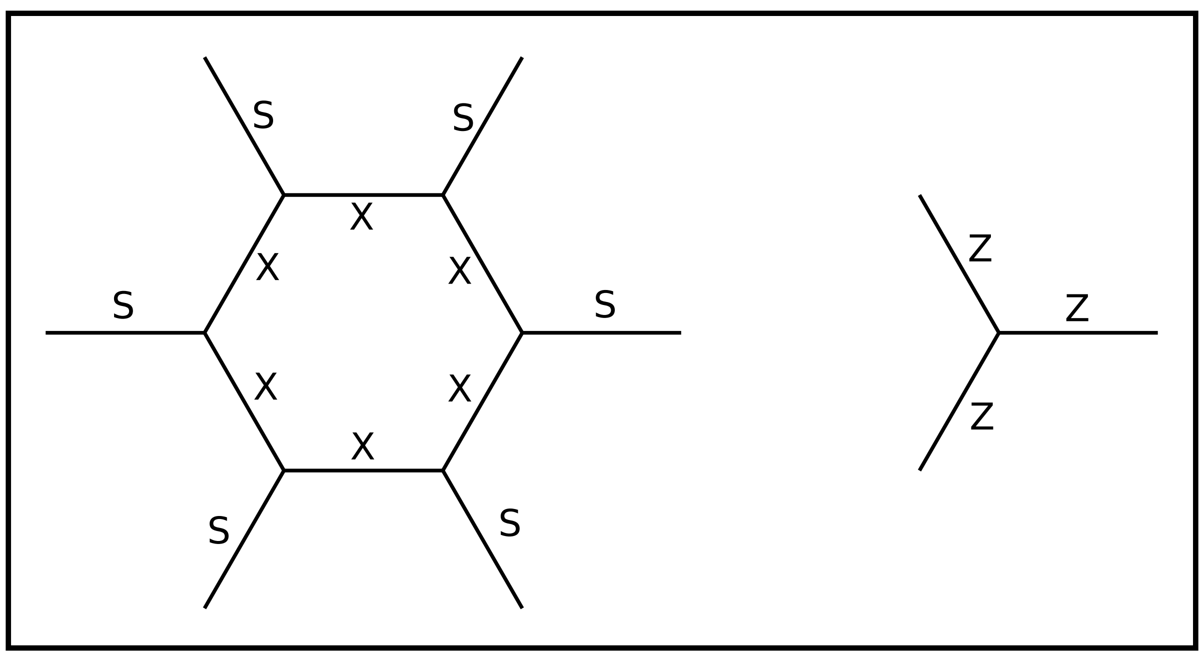
- Practical way to build an active quantum memory

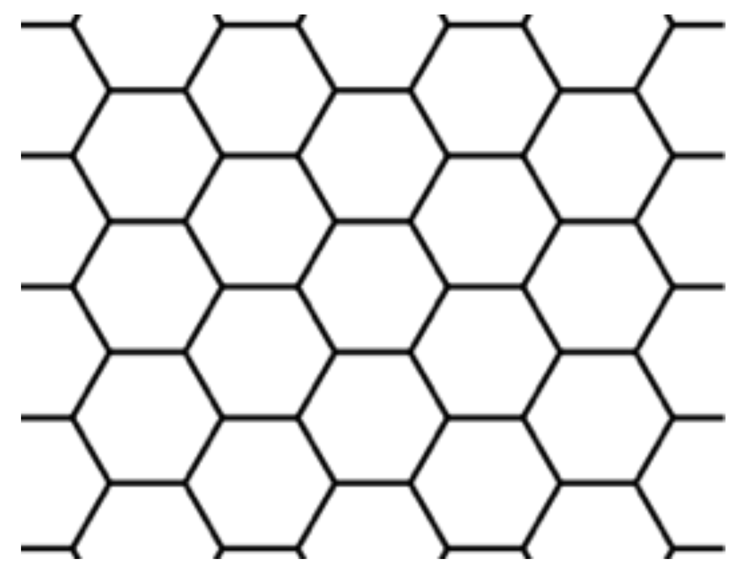
Toric (surface) code

- Practical way to build an active quantum memory
- Great example to understand basic properties of systems with topological order
  - Exactly solvable and simple
  - Contains features like anyons, string operators, boundary, twist, etc.

Bravyi Kitaev 98'
Bombin 10'

XS-stabilizer: double semion and more





Other motivations

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- (Efficient) representation of a larger class of quantum states

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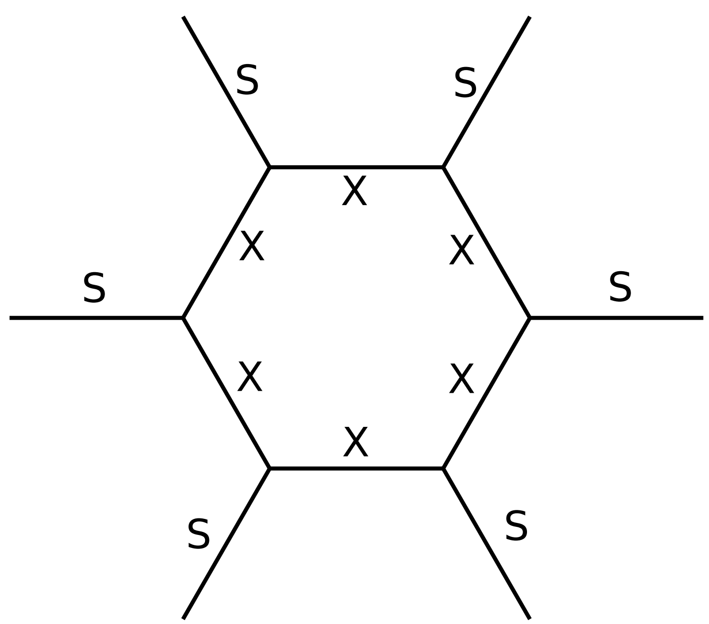
- (Efficient) representation of a larger class of quantum states
- A class of commuting projector problems that are in **NP** (**P**)

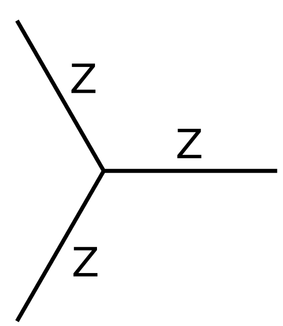
An introduction to the Double semion model

Double semion model

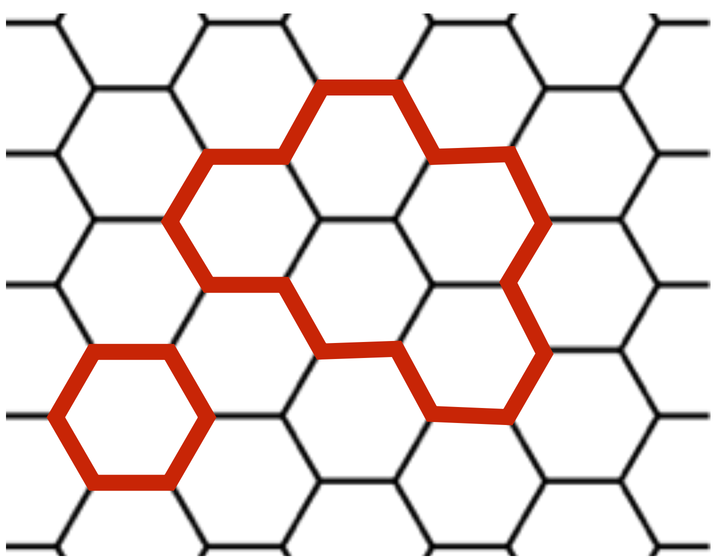
$$\sum_{x \text{ is close loops}} (-1)^{\text{number of loops}} |x\rangle$$



g_p



g_v

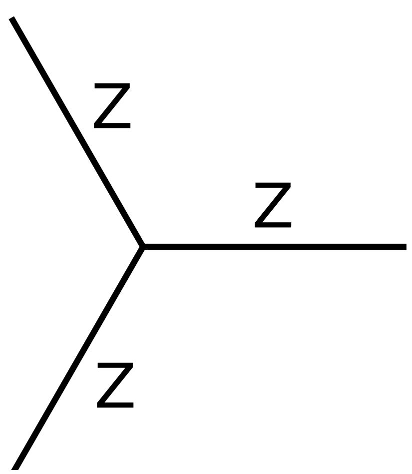


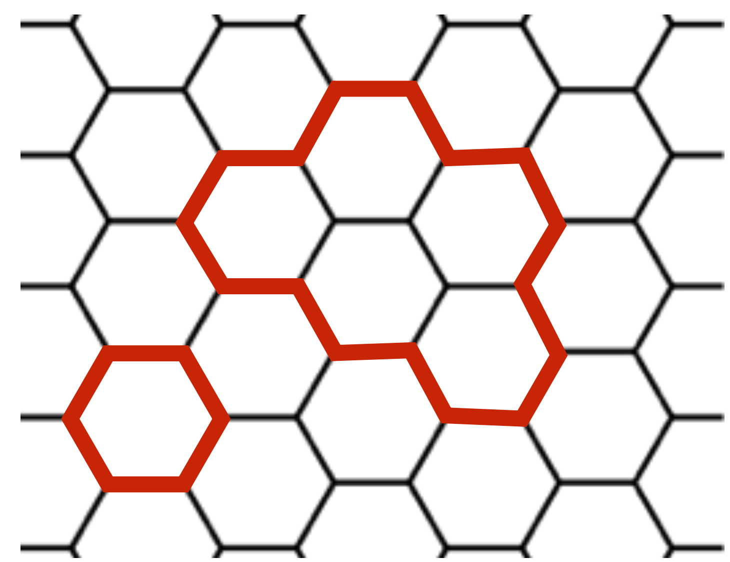
$|0\rangle$ $|1\rangle$

$$g_p|\psi\rangle = g_v|\psi\rangle = |\psi\rangle$$

Levin, Wen 05'

Z-type operator



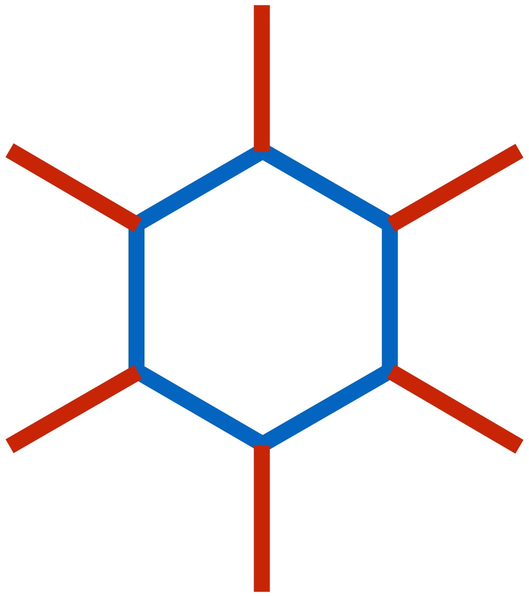


$|0\rangle$ $|1\rangle$

Gauge invariant subspace

Plaquette operator

x
s
z



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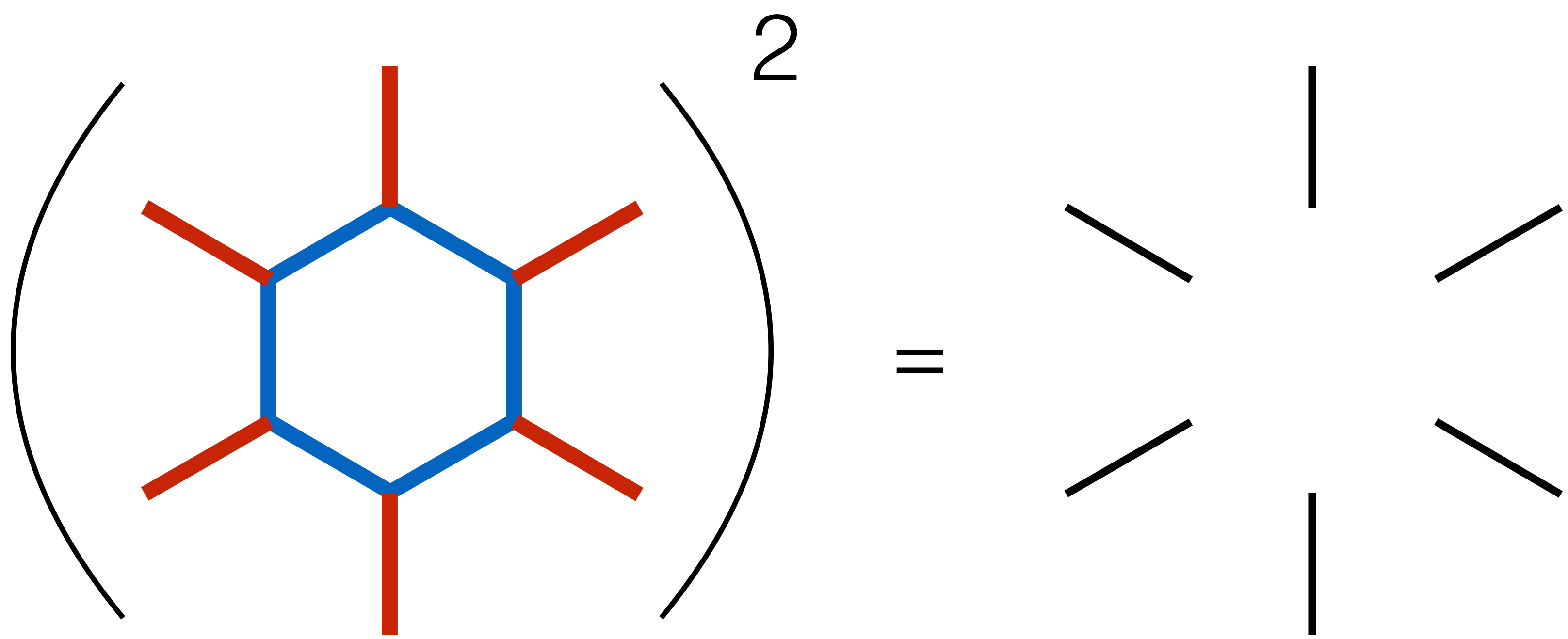
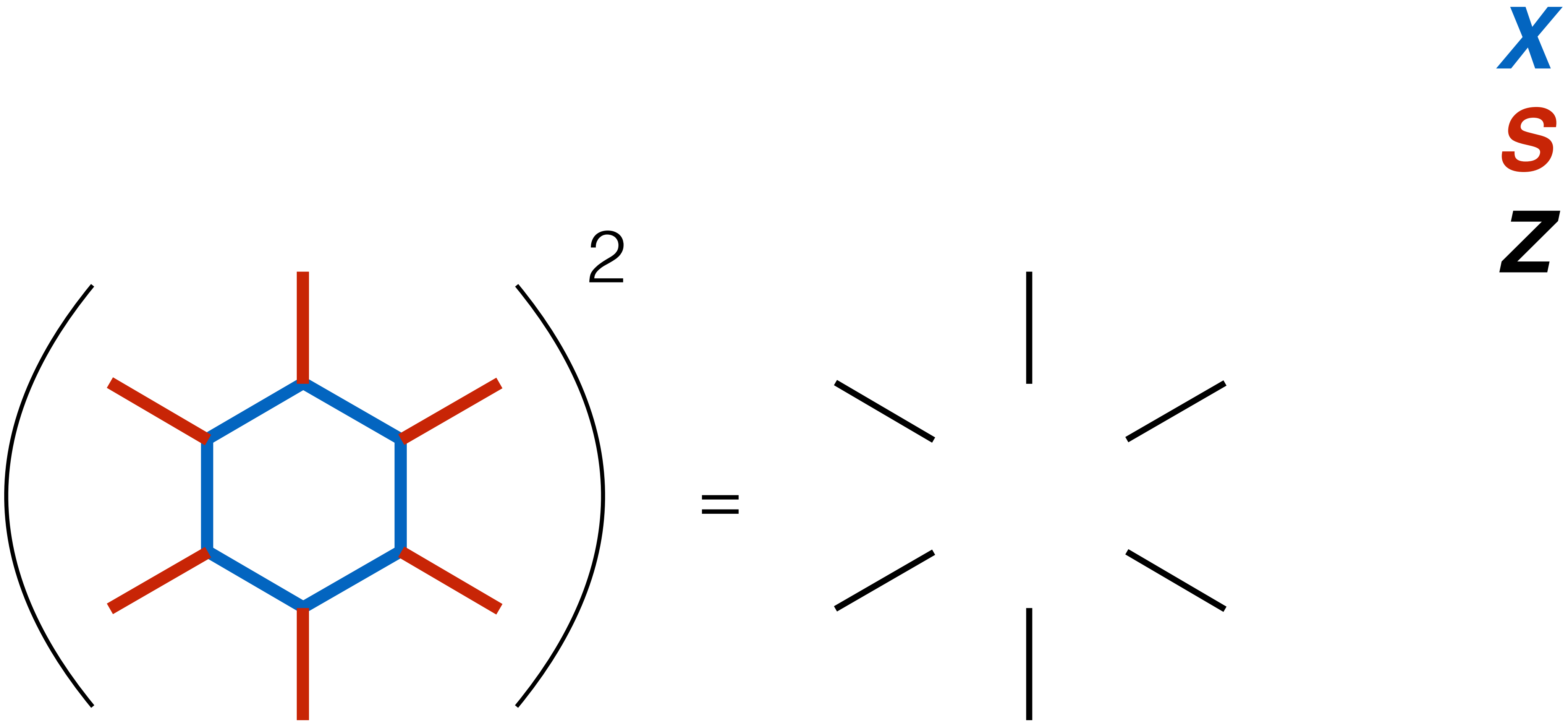
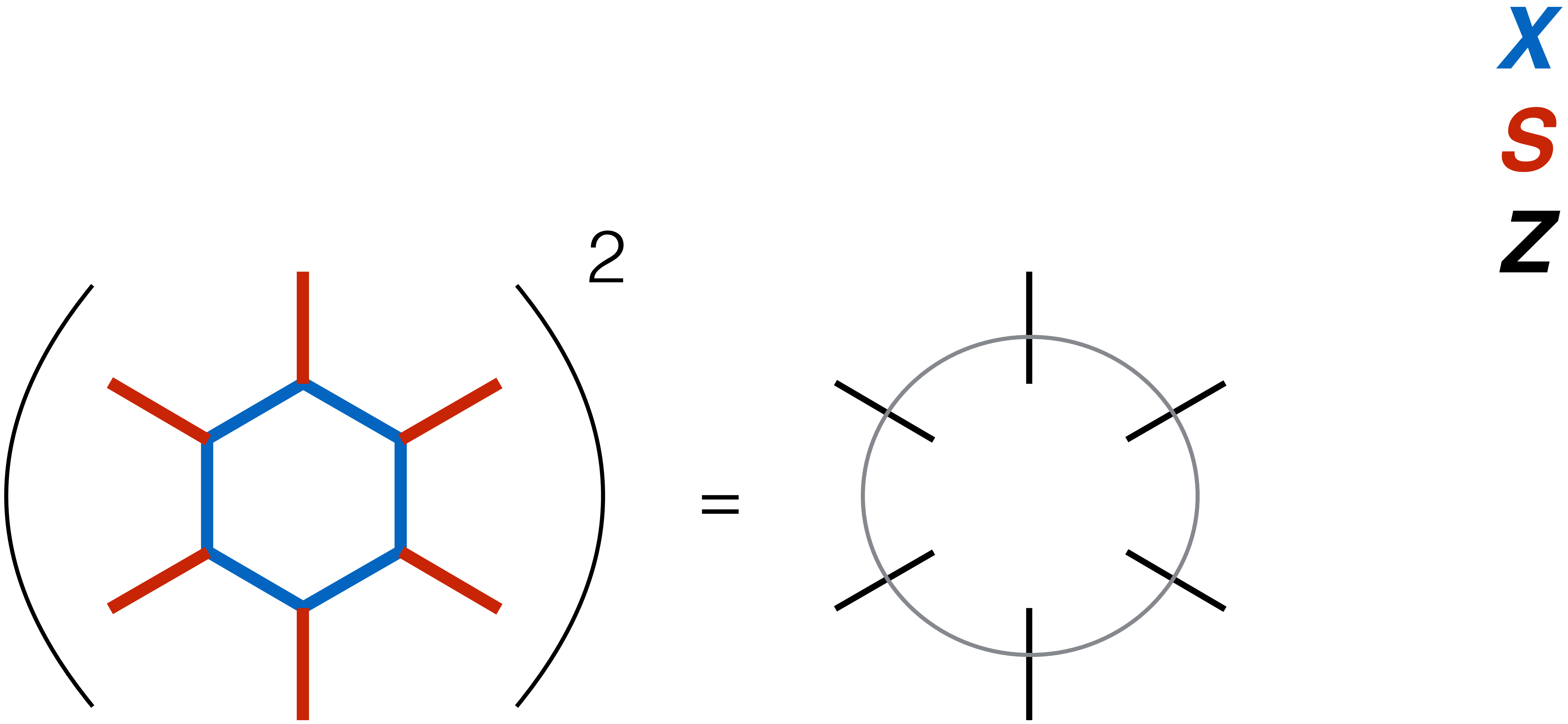


Plaquette operator



- The square is equal to 1 inside gauge invariant subspace

Plaquette operator



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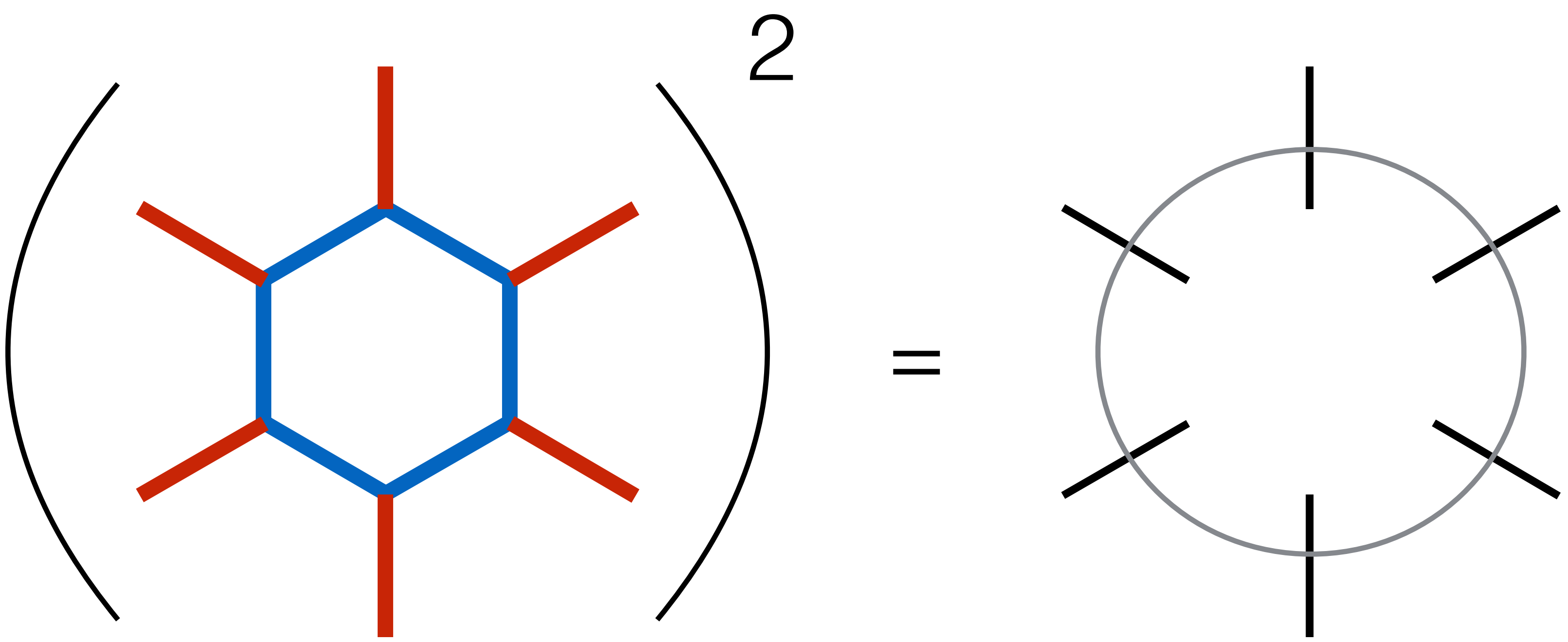

- The square is equal to 1 inside gauge invariant subspace
- Eigenvalue of original operator is ±1 inside the subspace

Commutator

$$[X,S]=XSX^{-1}S^{-1}=iZ$$

X
S
Z
XS
XS^3

Commutator

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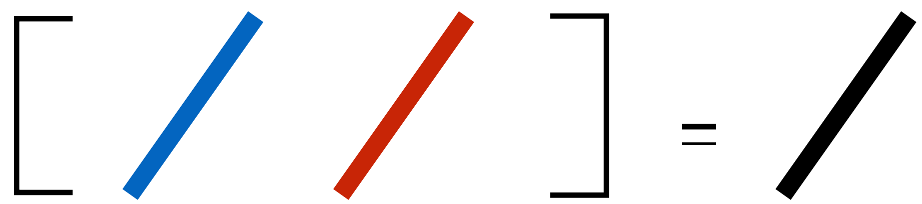

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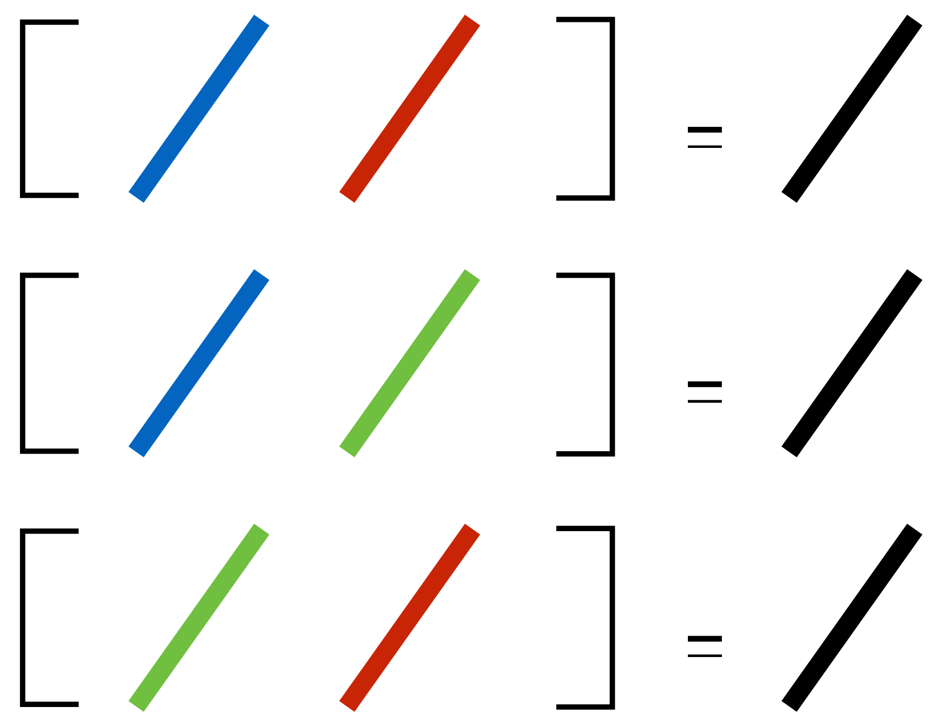

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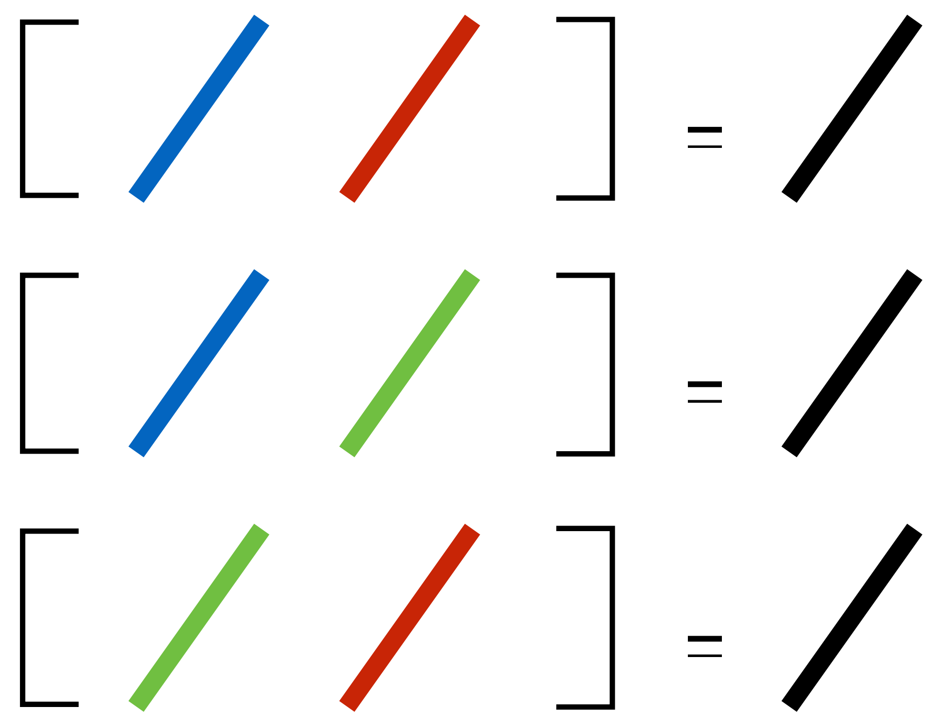

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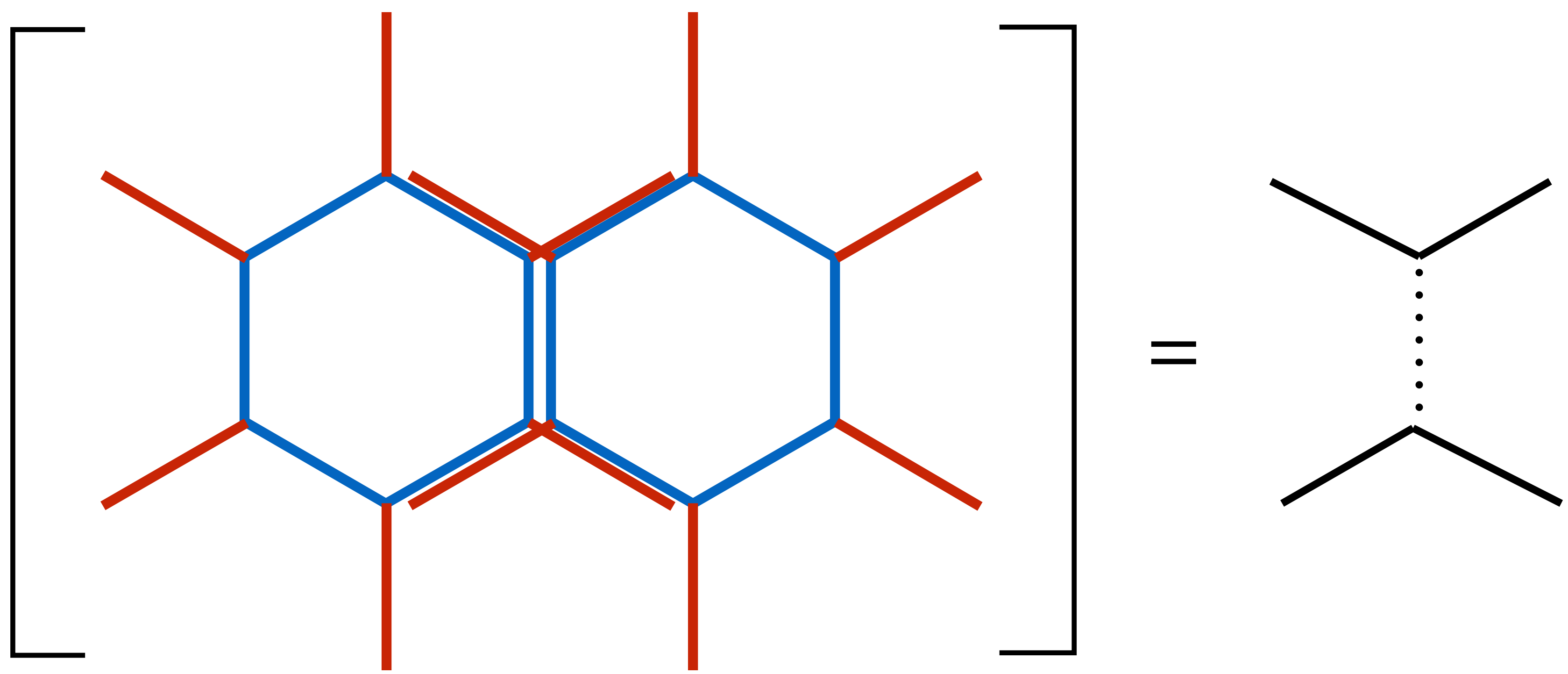
Different non-black color: ***Z***

Commutator



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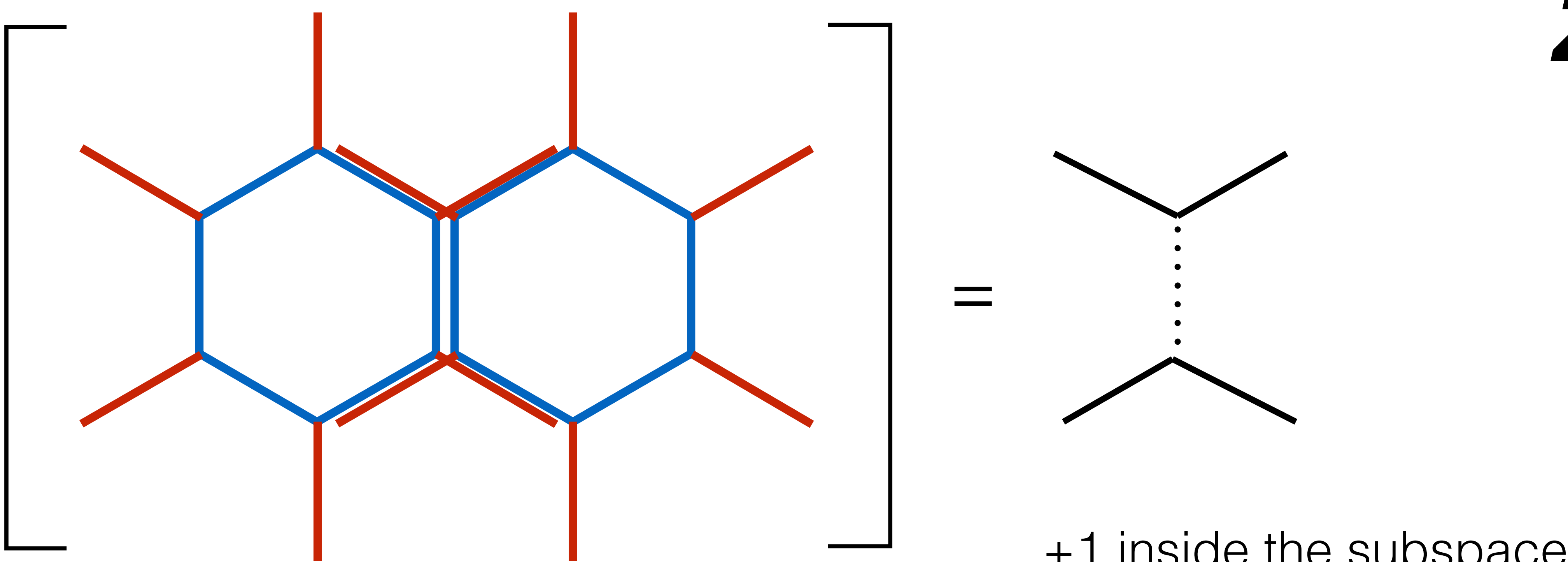

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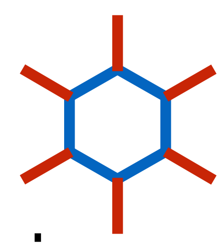
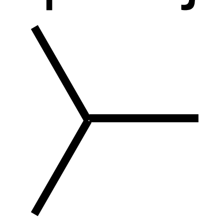
Commuting Hamiltonians

X
S
Z

- The Plaquette operators  are hermitian and commuting in the gauge invariant subspace

- The gauge invariant subspace = locally project into the +1 eigenspace of Z-type operators 

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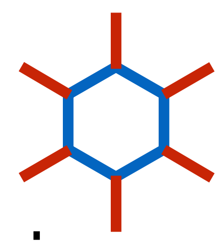
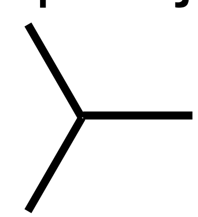
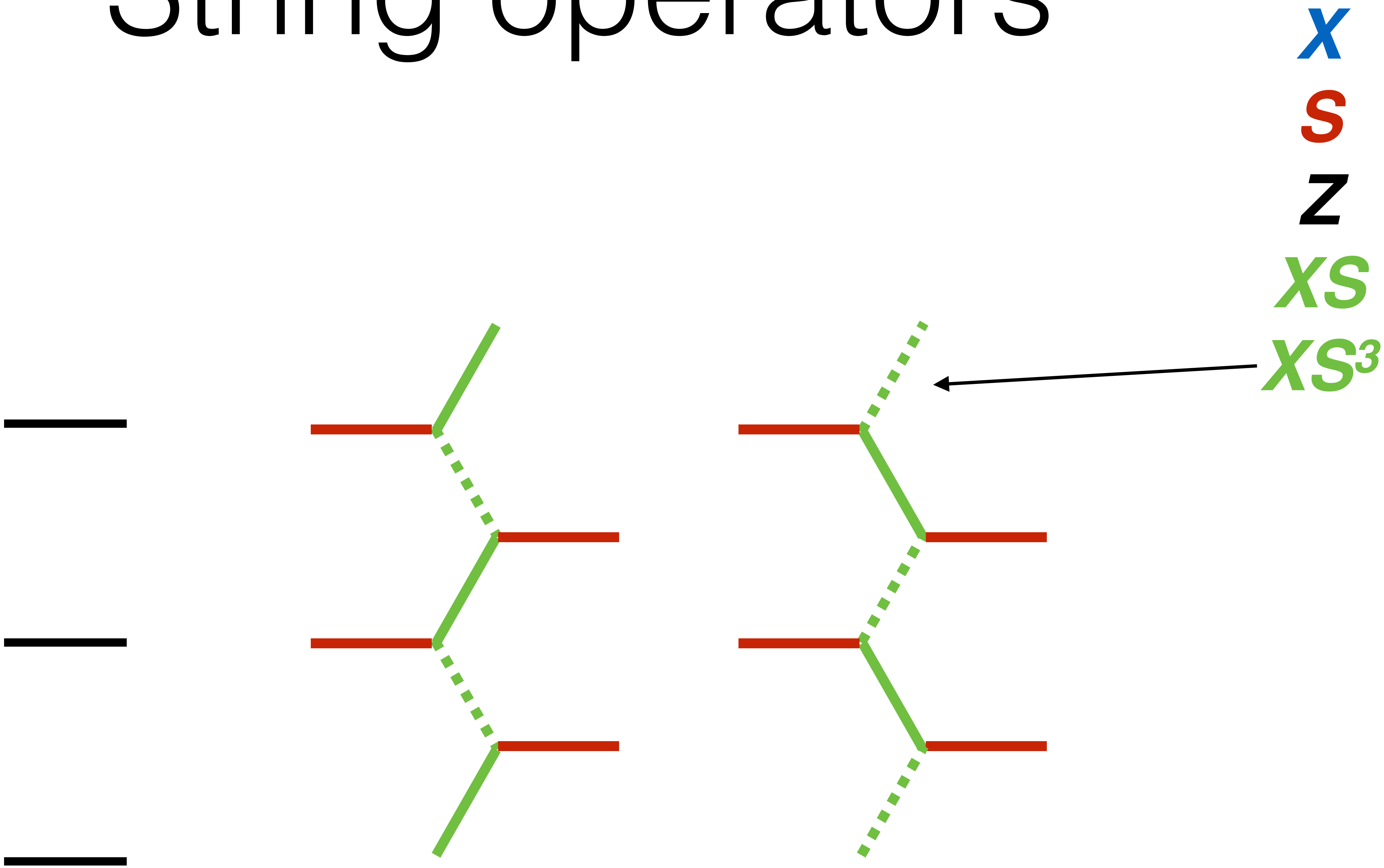
- The Plaquette operators  are hermitian and commuting in the gauge invariant subspace

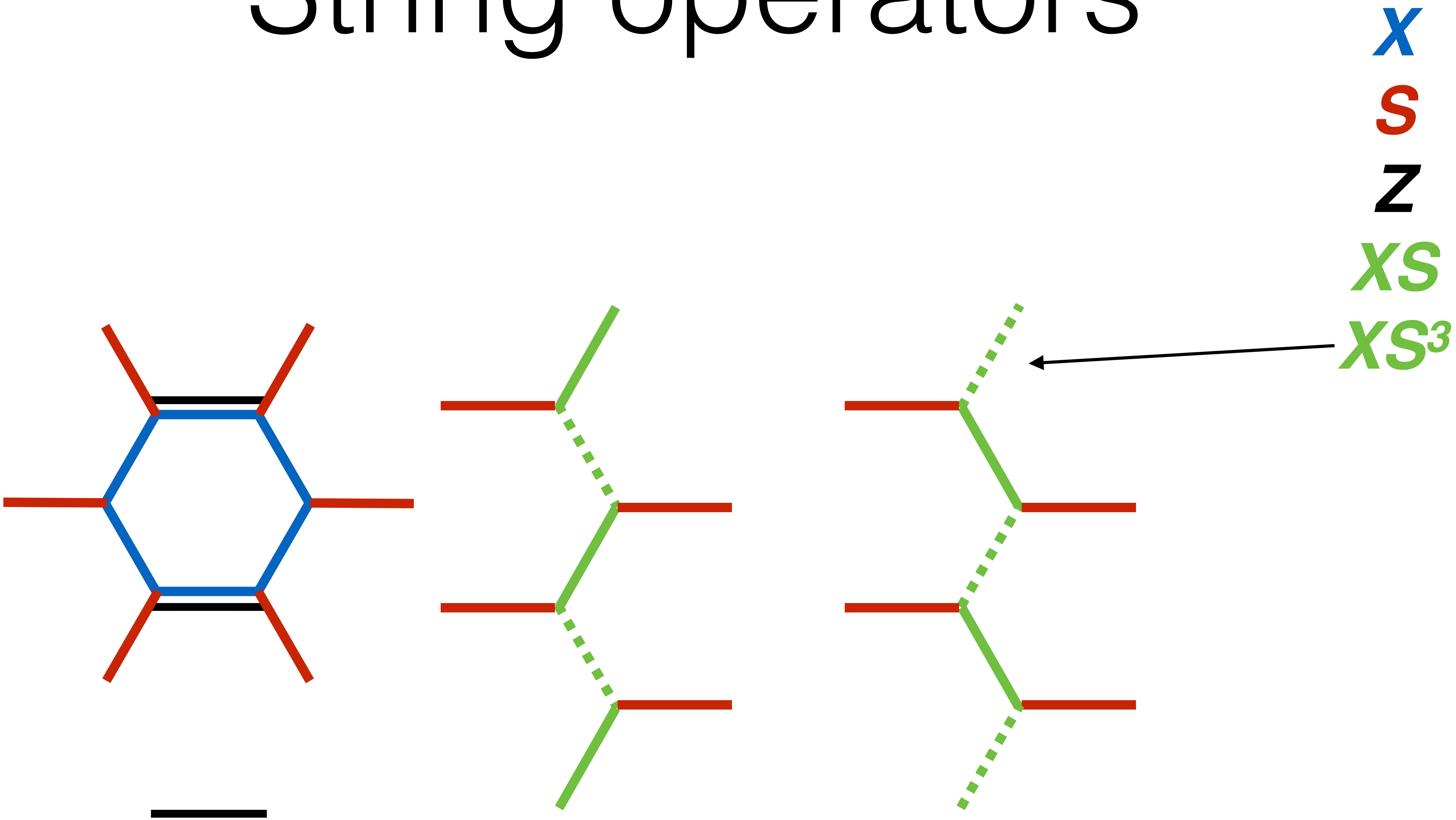
- The gauge invariant subspace = locally project into the +1 eigenspace of Z-type operators 

This is a general procedure!

String operators



String operators



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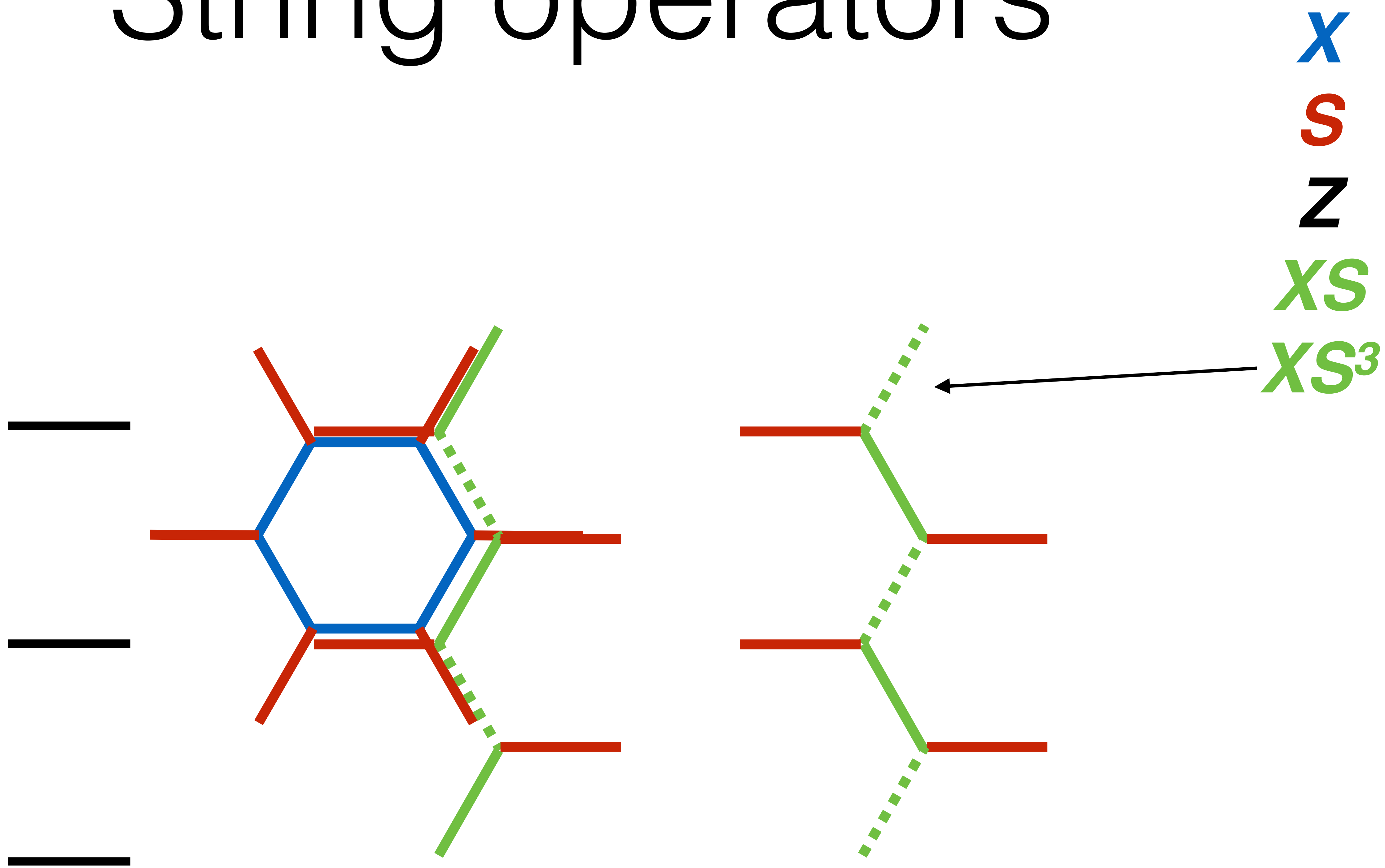

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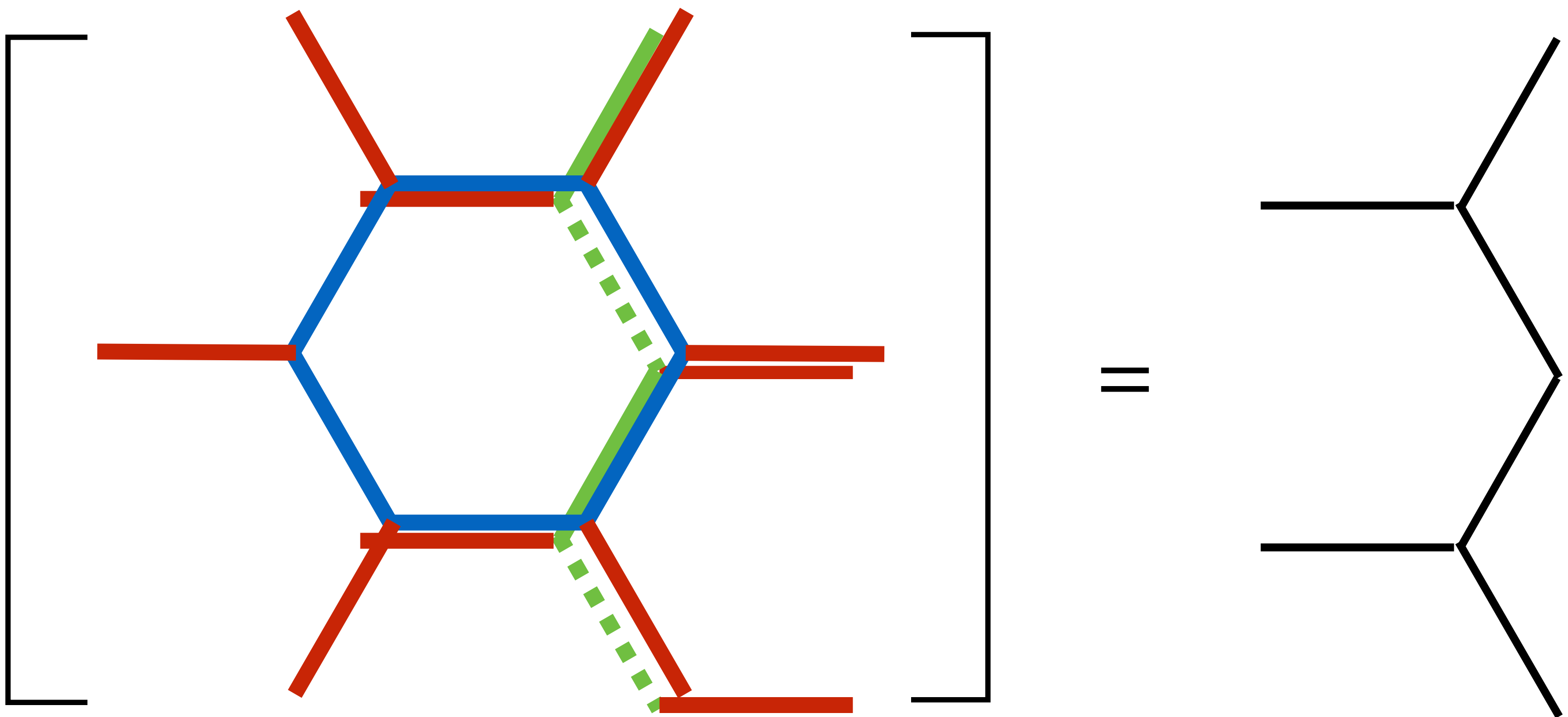
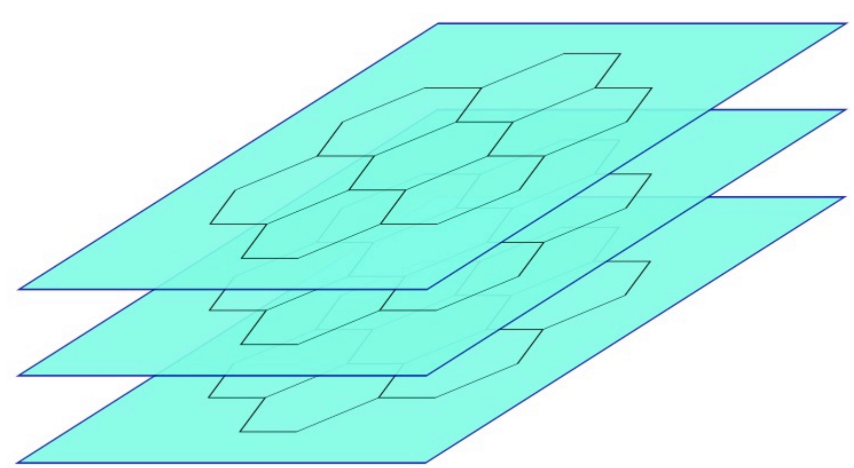


Different non-black color: ***Z***

Twisted quantum double

- Closed loops on each layer, with a phase add to each configuration

- (A subclass) can be described by XS stabilizer. Some of them support non-abelian anyons.



Hu, Wan, Wu 2012

Summary of properties

Computational complexity

- Given $G = \langle g_1, \ldots, g_m \rangle \subset \mathcal{P}_n^s$, is there a state stabilized by it?

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$$i^3 S_j \otimes S_k \otimes S_l$$

$$\vdots$$

NP-complete

1 in 3 SAT

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Diagonal stabilizers have no ***S***

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Degeneracy 2^k

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$\vdots$

NP-complete

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Double semion

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Form of the state

- We can construct a specific basis $\{\psi_j\}$ for the code space.

- For each ψ_j, we can efficiently find a circuit of (first) Clifford and (then) $\{T, CS, CCZ\}$ which generate the state

- $\langle\psi_j|P|\psi_k\rangle$ can be computed for Pauli operator P efficiently.

Entanglement property

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- For a given XS-stabilizer state $|\psi\rangle$ and a bipartition (A, B), we can efficiently find a Pauli state $|\varphi_{AB}\rangle$ and $U_A \otimes U_B$ such that $U_A \otimes U_B|\psi\rangle = |\varphi_{AB}\rangle$.

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- Indirect way to compute entropy for XS states.

- Reflects the fact that toric code and double semion have very similar entanglement properties.

Flammia et al. 09

Outlooks

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- Understanding entanglement properties better

Thanks

Sydney modern art museum